

Paper Reference 9FM0/3C
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
PAPER 3C: Further Mechanics 1

Wednesday 14 June 2023 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical
Tables (Green), calculator**

YOU WILL BE GIVEN

Diagram Booklet
Answer Booklet

Y72798A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 7 questions in this question paper. The total mark for this paper is 75.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. A particle **P** of mass **2 kg** is moving with velocity $(-4\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$ when it receives an impulse $(-6\mathbf{i} + 42\mathbf{j}) \text{ N s}$

- (a) Find the speed of **P** immediately after receiving the impulse.
(4 marks)

(continued on the next page)

1. continued.

The angle through which the direction of motion of P has been deflected by the impulse is α°

**(b) Find the value of α
(2 marks)**

(Total for Question 1 is 6 marks)

Turn over

2. A car of mass **1000 kg** moves in a straight line along a horizontal road at a constant speed **$U \text{ m s}^{-1}$**

The resistance to the motion of the car is a constant force of magnitude **400 newtons**.

The engine of the car is working at a constant rate of **16 kW**

- (a) Find the value of **U**
(3 marks)

(continued on the next page)

Turn over

2. continued.

The car now pulls a trailer of mass 600 kg in a straight line along the road using a tow rope which is parallel to the direction of motion.

The resistance to the motion of the car is again a constant force of magnitude 400 newtons.

The resistance to the motion of the trailer is a constant force of magnitude 300 newtons.

The engine of the car is working at a constant rate of 16 kW

(continued on the next page)

Turn over

2. continued.

The tow rope is modelled as being light and inextensible.

Using the model,

- (b) find the tension in the tow rope at the instant when the speed of the car is $\frac{20}{3} \text{ ms}^{-1}$ (5 marks)**

(Total for Question 2 is 8 marks)

Turn over

3. A particle **P** of mass **$2m$** is moving in a straight line with speed **$3u$** on a smooth horizontal plane.

It collides directly with a particle **Q** of mass **m** that is moving on the plane with speed **$2u$** in the opposite direction to **P**

(continued on the next page)

3. continued.

The coefficient of restitution between P and Q is e , where $e > \frac{4}{5}$

(a) Show that the speed of Q immediately after the collision is

$$\frac{(4 + 10e)u}{3}$$

(6 marks)

(continued on the next page)

Turn over

3. continued.

After the collision Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q

The coefficient of restitution between Q and the wall is f

(b) Find, in terms of e , the set of values of f for which there will be a second collision between P and Q

(4 marks)

(Total for Question 3 is 10 marks)

Turn over

4. A light elastic string has natural length $2a$ and modulus of elasticity $4mg$

One end of the elastic string is attached to a fixed point O

A particle P of mass m is attached to the other end of the elastic string.

The particle P hangs freely in equilibrium at the point E , which is vertically below O

- (a) Find the length OE
(4 marks)

(continued on the next page)

Turn over

4. continued.

Particle **P** is now pulled vertically downwards to the point **A**, where $OA = 4a$, and released from rest.

The resistance to the motion of **P** is a constant force of magnitude $\frac{1}{4} mg$

- (b) Find, in terms of **a** and **g**, the speed of **P** after it has moved a distance **a**
(7 marks)

(continued on the next page)

Turn over

4. continued.

Particle **P** is now held at **O**

Particle **P** is released from rest and reaches its maximum speed at the point **B**

The resistance to the motion of **P** is again a constant force of magnitude $\frac{1}{4} mg$

(c) Find the distance **OB**
(4 marks)

(Total for Question 4 is 15 marks)

Turn over

5. Refer to the diagram for Question 5 in the Diagram Booklet.

A smooth uniform sphere **S** of mass **m** is moving with speed **U** on a smooth horizontal plane.

The sphere **S** collides obliquely with another uniform sphere of mass **M** which is at rest on the plane.

The two spheres have the same radius.

(continued on the next page)

Turn over

5. continued.

Immediately before the collision the direction of motion of **S** makes an angle α , where $0 < \alpha < 90^\circ$, with the line joining the centres of the spheres.

Immediately after the collision the direction of motion of **S** makes an angle β with the line joining the centres of the spheres, as shown in the diagram.

(continued on the next page)

Turn over

5. continued.

The coefficient of restitution between the spheres is e

(a) Show that

$$\tan \beta = \frac{(m + M) \tan \alpha}{(m - eM)}$$

(8 marks)

(continued on the next page)

Turn over

5. continued.

Given that

$$\mathbf{m = eM,}$$

(b) show that the directions of motion of the two spheres immediately after the collision are perpendicular.

(2 marks)

(Total for Question 5 is 10 marks)

Turn over

6. A particle **P** of mass **m** is falling vertically when it strikes a fixed smooth inclined plane.

The plane is inclined to the horizontal at an angle α , where $0 < \alpha \leq 45^\circ$

At the instant immediately before the impact, the speed of **P** is **u**

At the instant immediately after the impact, **P** is moving horizontally with speed **v**

(continued on the next page)

6. continued.

(a) Show that the magnitude of the impulse exerted on the plane by P is $\mu \sec \alpha$

(5 marks)

The coefficient of restitution between P and the plane is e , where $e > 0$

(b) Show that

$$v^2 = u^2 (\sin^2 \alpha + e^2 \cos^2 \alpha)$$

(3 marks)

(continued on the next page)

Turn over

6. continued.

(c) Show that the kinetic energy lost by P in the impact is

$$\frac{1}{2} mu^2 (1 - e^2) \cos^2 \alpha$$

(2 marks)

(d) Hence find, in terms of m , u and e ONLY, the kinetic energy lost by P in the impact.

(2 marks)

(Total for Question 6 is 12 marks)

Turn over

- 7. Refer to the diagram for Question 7 in the Diagram Booklet.**

A small smooth snooker ball is projected from the corner *A* of a horizontal rectangular snooker table *ABCD*

The ball is projected so it first hits the side *DC* at the point *P*, then hits the side *CB* at the point *Q* and then returns to *A*

(continued on the next page)

Turn over

7. continued.

Angle $APD = \alpha$, Angle $QPC = \beta$,

Angle $AQB = \gamma$

The ball moves along AP with speed U , along PQ with speed V and along QA with speed W , as shown in the diagram.

The coefficient of restitution between the ball and side DC is e_1

The coefficient of restitution between the ball and side CB is e_2

(continued on the next page)

Turn over

7. continued.

The ball is modelled as a particle.

Use the model to answer all parts of this question.

**(a) Show that $\tan \beta = e_1 \tan \alpha$
(4 marks)**

**(b) Hence show that
 $e_1 \tan \alpha = e_2 \cot \gamma$
(3 marks)**

(continued on the next page)

Turn over

7. continued.

- (c) By considering (angle APQ + angle AQP) or otherwise, show that it would be possible for the ball to return to A only if $e_2 > e_1$ (6 marks)**

(continued on the next page)

Turn over

7. continued.

If instead $\mathbf{e}_1 = \mathbf{e}_2$, the ball would
NOT return to A

Given that $\mathbf{e}_1 = \mathbf{e}_2$

(d) use the result from part (b) to
describe the path of the ball after
it hits **CB** at **Q**, explaining your
answer.

(1 mark)

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
